CIBM
Center for Biomedical Imaging

BASICS OF MRI

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PhD Course, 09 of Octobre 2020

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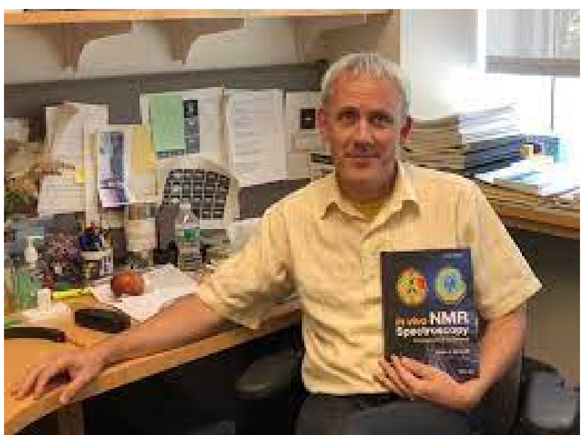
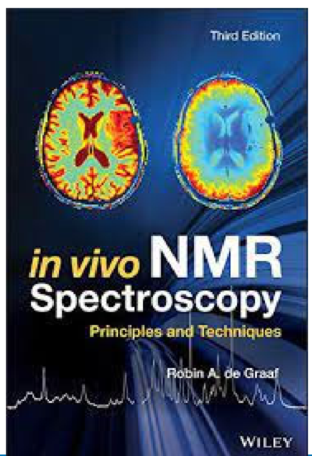
OUTLINE

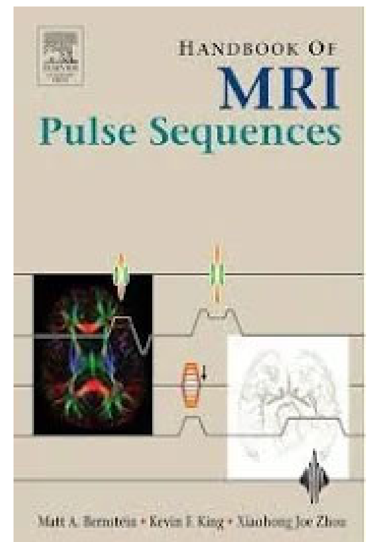
- Localization - Magnetic field gradients – gradient coils
- Spatial encoding
  - Slice selection
  - Frequency encoding
  - Phase encoding
- How to encode a 3D image
- Some notions about k-space
  - Discrete Fourier transform
  - What is where in k-space
  - How to fill the k-space

FURTHER READING

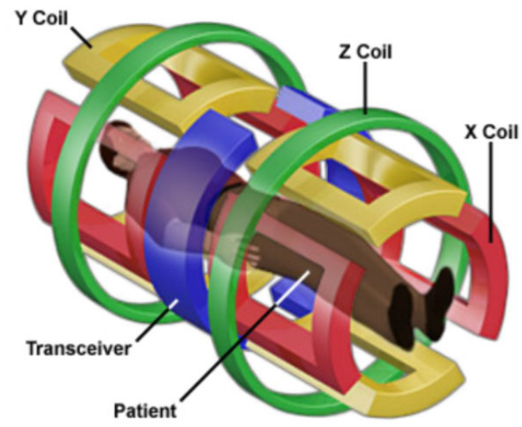
- [FundBioImag – YouTube]
- [Basics of In Vivo NMR - YouTube]







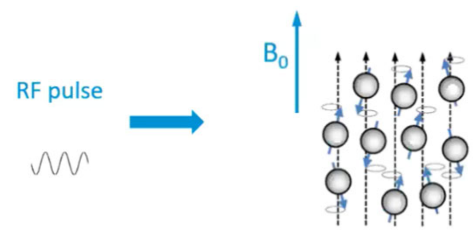
MAGNETIC FIELD GRADIENTS – GRADIENT COILS

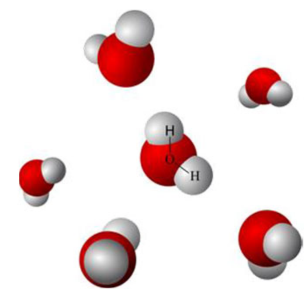


UNTIL NOW

Most MRI applications measure 1H nuclei from water

- **Strong static field (B_0)**
- **RF pulses**
- **FID – relaxation**
  - M_{xy} decays, M_z grows (T_2 and T_1 relaxation)
- **No spatial information - RF coils measure signal from entire body**

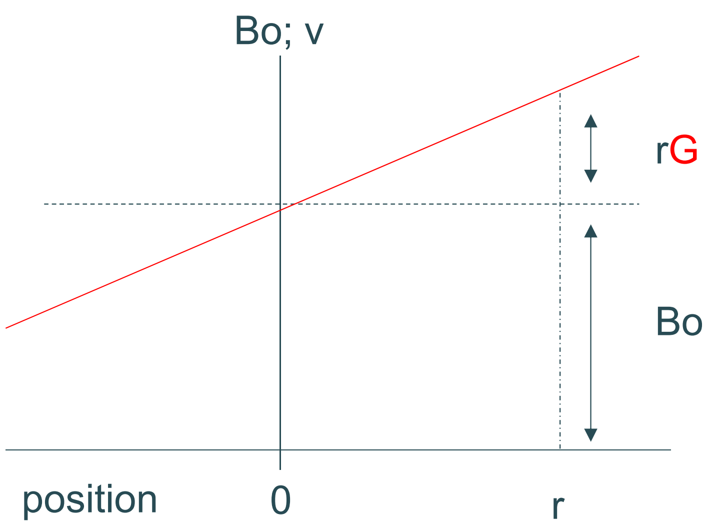




Precession Frequency $\omega_L=\gamma B_0$ Static Magnetic Field

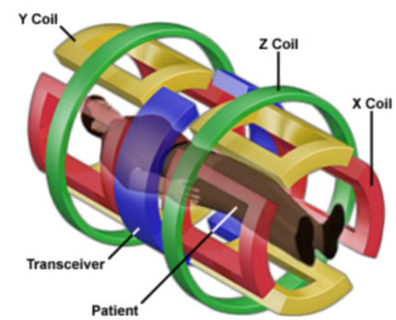
HOW TO ENCODE SPATIAL POSITION ?

Magnetic field gradients - differentiate the signal across space



Magnetic field B along z
varies spatially with x, y, and/or z:

$$\vec{G} \equiv \frac{dB_z}{d\vec{r}}$$



$$\boxed{\nu = \gamma B_0}$$

External magnetic field B_0 = 3.0 Tesla (T) = 30,000 Gauss (G)
Gyromagnetic ratio $\gamma(^1H)$ = 42.57 MHz/T = 4257 Hz/G
Larmor frequency ν = 127.7 MHz

$$\boxed{\nu = \gamma B_0 + \gamma r G}$$

Position r = 0.04 meters (m) = 4 cm
Gradient G = 0.05 T/m = 5 G/cm = 21.285 kHz/cm
Additional frequency offset = 85.14 kHz

$$\boxed{\phi \text{ (radians)} = 2\pi\nu t}$$

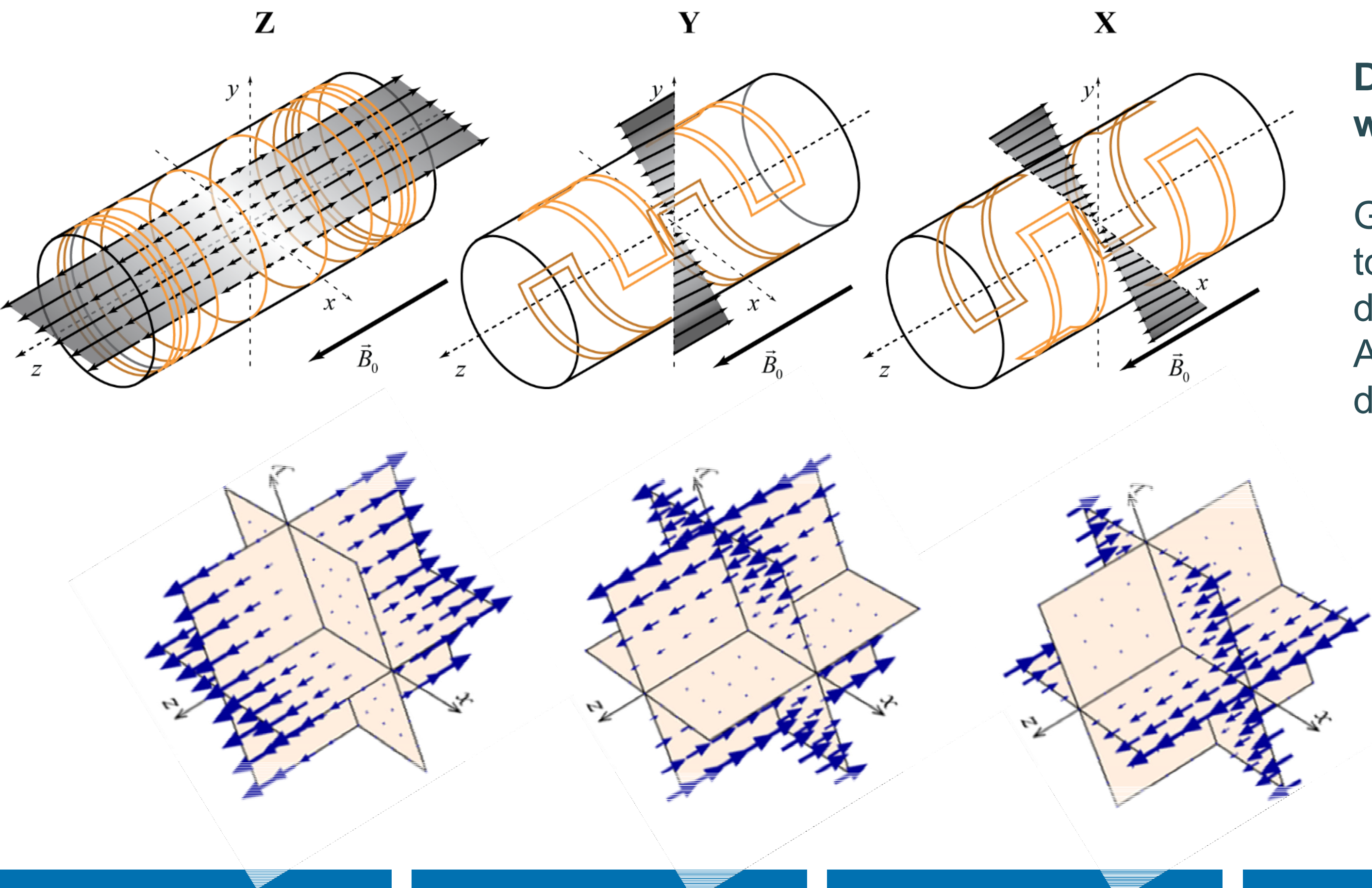
GRADIENT COILS

- Gradient coil generate a magnetic field along the main B_0 field varying linearly along one axis (x, y or z):
  - $\vec{G}_x = x \cdot G_x \cdot \vec{e}_z$
  - $\vec{G}_y = y \cdot G_y \cdot \vec{e}_z$
  - $\vec{G}_z = z \cdot G_z \cdot \vec{e}_z$
- By applying a (magnetic) gradient, we modify the resonance frequency locally:
  - $\omega = \gamma \cdot B_0 + \gamma \cdot x \cdot G_x = \omega_0 + \underbrace{\gamma \cdot x \cdot G_x}_{\delta\omega}$

Or more generally if multiple gradients are applied at the same time

  - $\omega = \omega_0 + \gamma \cdot \left(x \cdot G_x + y \cdot G_y + z \cdot G_z\right)$

GRADIENT COILS



Direction of gradient = direction in which the magnetic filed varies

Gradient coils generate B_0 field parallel to B_0 – add B_0 to main B_0 which is on Z direction

Amount of B_0 depends linearly with direction

GRADIENT COILS

- Amplitude G/cm or mT/m (10.000G=1T)
- Linearity (homogeneity)
- Rise time (0 to max amplitude, 120 µs) – slew rate=G_{max}/T_{rise}
- Change current in B_0 – Lorentz force – noise in MRI
- Eddy currents – how we correct for them

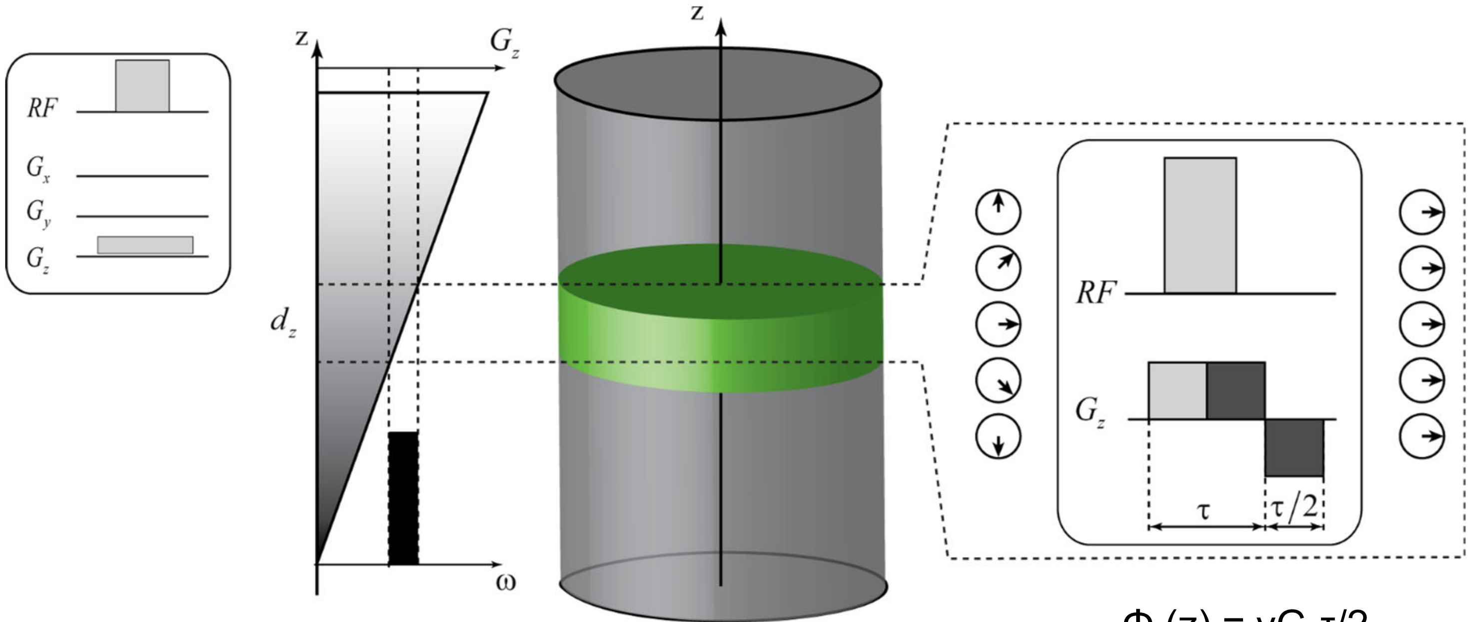
SPATIAL ENCODING

SPATIAL ENCODING

- MR relies on four different methods to spatially encode the NMR signal
- Gradient based methods:
  - Frequency Encoding (73' Lauterbur, 1973 Mansfield)
  - Slice selection (74' Mansfield)
  - Phase encoding (76' Ernst)
- Geometrical method:
  - RF-coil sensitivity profile (97' Sodickson, 99' Pruessman, 01' Larkman,…)

[1]Vilstrup et al., Hepathology 2014;60

SPATIAL ENCODING – SLICE SELECTION



$$\delta z = \frac{BW_{RF}}{\gamma G_z}$$

$$\Phi\ (z) = \gamma G_z \tau / 2$$

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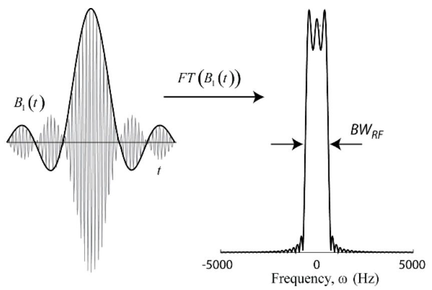
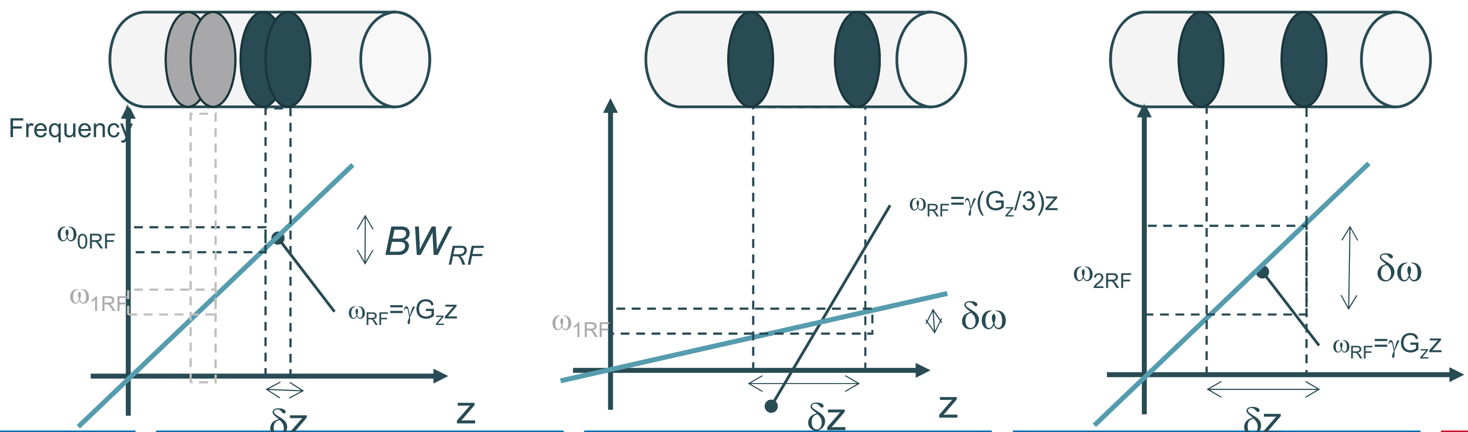
SLICE SELECTION

- Gradient is applied during an RF-pulse, which is characterized by its excitation bandwidth (BW_{RF})

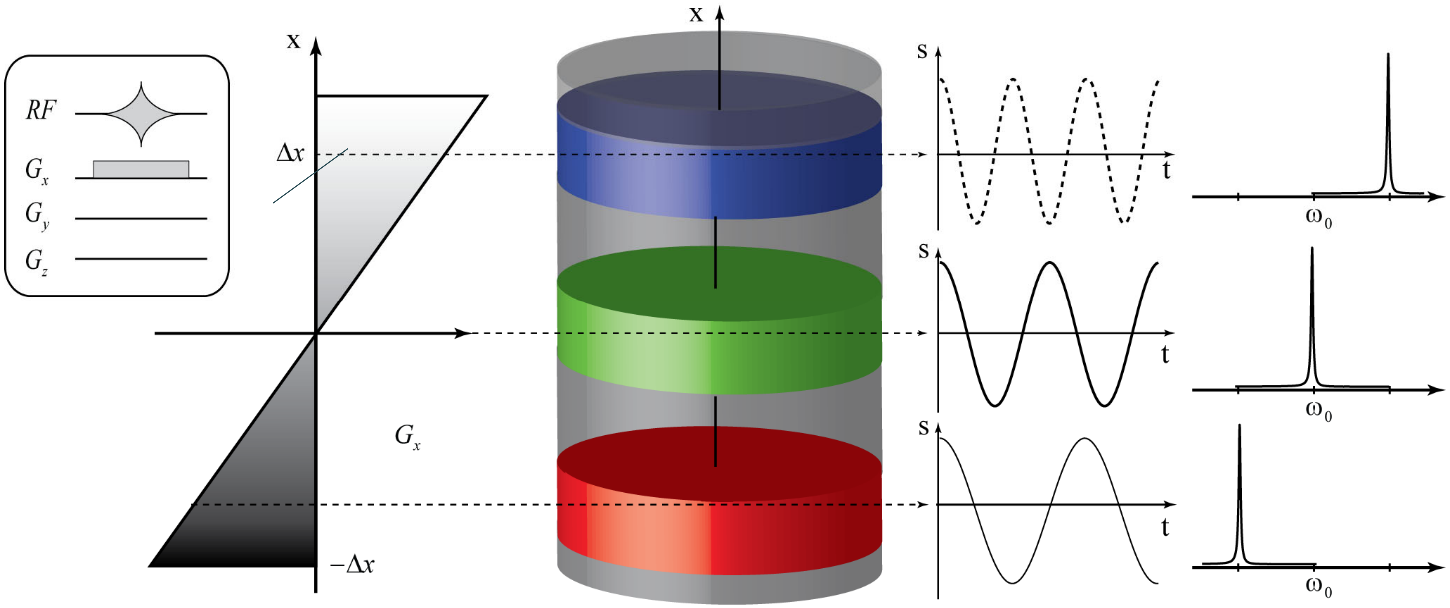


$$\delta z = \frac{BW_{RF}}{\gamma G_z}$$

- The slice thickness can be changed either by adjusting the pulse length or the gradient amplitude



SPATIAL ENCODING – FREQUENCY ENCODING



$$S(t) = \int f(x) \cdot e^{-t/T2*} \cdot e^{-i\omega(x)t} dx$$

$$S = e^{-t/T2*} \cdot e^{-i2\pi\gamma B_0 t}$$

$$\int f(x) \cdot e^{-i2\pi(\gamma G_x t x)} dx$$

$$S(t) = A \cdot \int f(x) \cdot e^{-i2\pi(k_x x)} dx$$

$$S = A \cdot F(kx = \gamma Gx\ t)$$

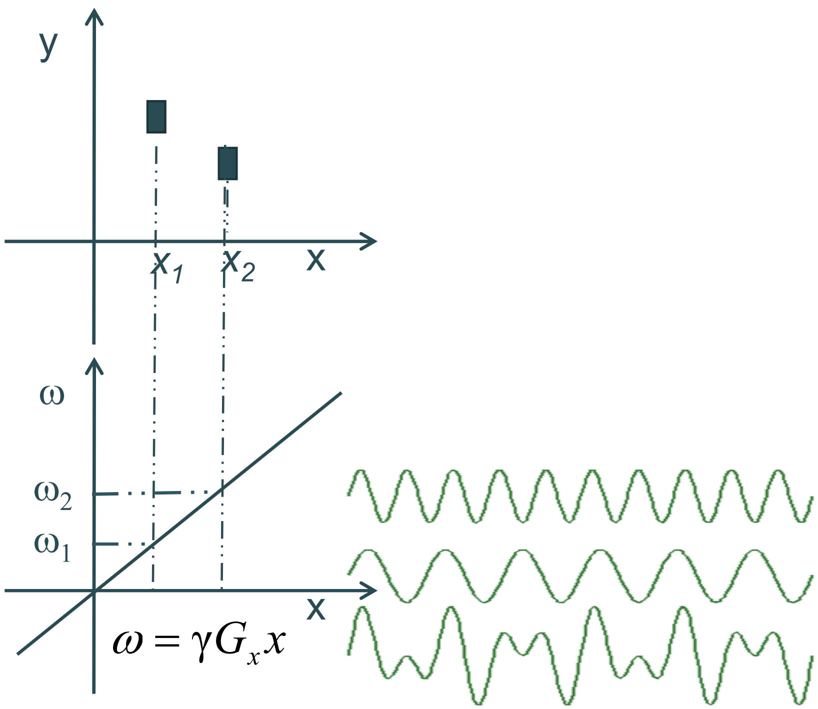
$$\vec{G}_x = x \cdot G \cdot \vec{e}_z$$

$$\omega = \gamma \cdot x \cdot G_x$$

$$dS(x,t) = f(x) \cdot e^{-t/T2*} \cdot e^{-i\omega(x)t}$$

FREQUENCY ENCODING

- Consider the simple 2-dimensional object, O(x,y)



$$S_1 = A_1 e^{-i\gamma G_x x_1 t} = A_1 e^{-i\omega_1 t}$$

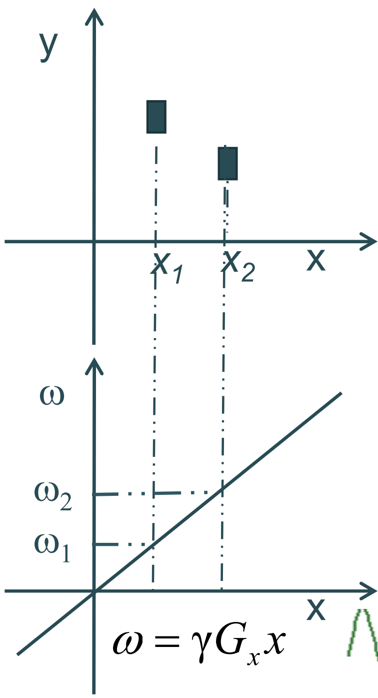
$$S_2 = A_2 e^{-i\gamma G_x x_2 t} = A_2 e^{-i\omega_2 t}$$

$$S(t) = S_1 + S_2 =$$

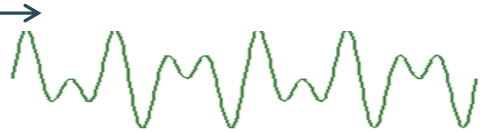
$$A_1 e^{-i\omega_1 t} + A_2 e^{-i\omega_2 t}$$

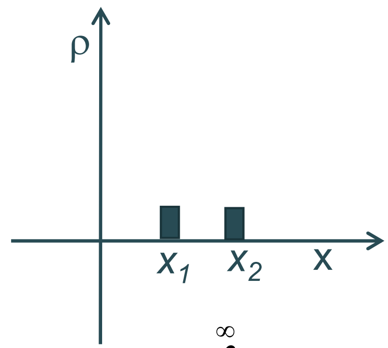
FREQUENCY ENCODING

- Consider the simple 2-dimensional object, O(x,y)



$$\omega = \gamma G_x x$$





$$\rho\left(x = \frac{\omega}{\gamma G_x}\right) = \int_{-\infty}^{\infty} S(t) e^{i\omega t} dt$$

Fourier Transform

$$S(t) = S_1 + S_2 =$$
$$A_1 e^{-i\omega_1 t} + A_2 e^{-i\omega_2 t}$$

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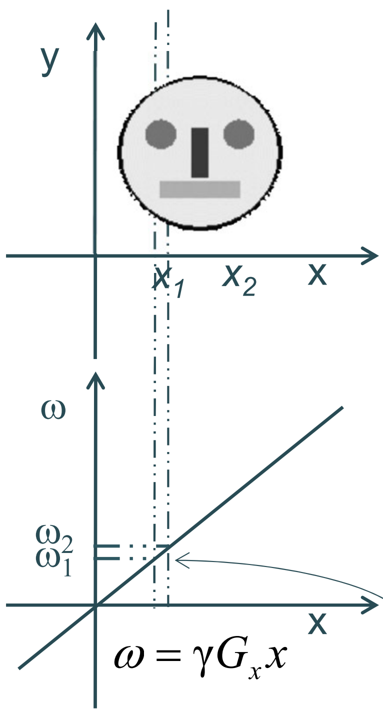
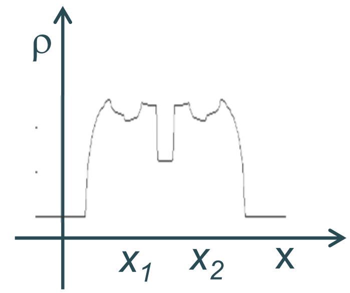
FREQUENCY ENCODING

– Consider a more general 2-dimensional object, O(x,y)



$$\omega = \gamma G_x x$$

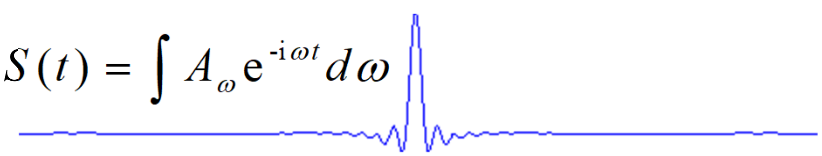
Fourier Transform of the Frequency encoded signal yields a projection of the object



$$FT[S(t = \tfrac{k_x}{\gamma G_x})] = \int_{-\infty}^{\infty} S(t) e^{i\omega t} dt = P(x = \tfrac{\omega}{\gamma G_x})$$

Fourier Transform

$$S(t) = \int A_\omega \mathrm{e}^{-i\omega t} d\omega$$

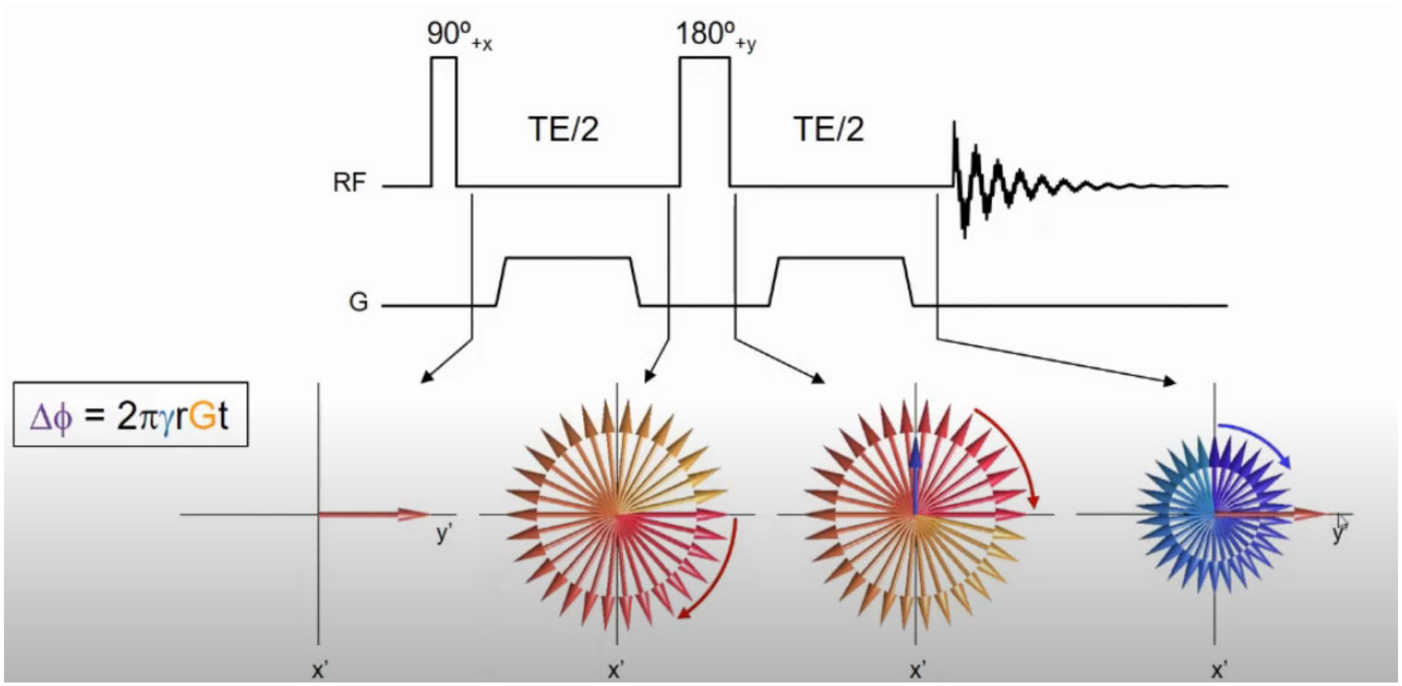


$$A_\omega d\omega = \left[\int M^+(x,y)dy\right]dx = P_y(x)dx$$

Projection of the object

C I B M . C H

MAGNETIC FIELD GRADIENT FOR SIGNAL CRUSHING



10. MR Hardware - Magnetic Field Gradients - YouTube

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FREQUENCY ENCODING – POLARITY – ECHO FORMATION

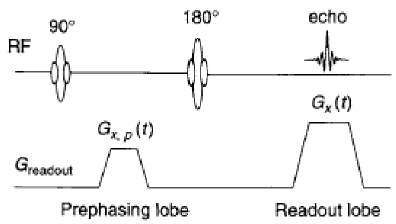


FIGURE 8.2 A frequency-encoding gradient waveform in a spin-echo pulse sequence. The waveform consists of two lobes; a prephasing gradient lobe $G_{x,p}(t)$ and a readout gradient lobe $G_x(t)$, both of which have the same polarity.

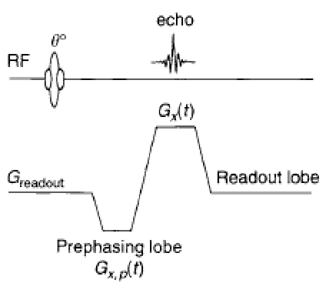


FIGURE 8.3 A frequency-encoding gradient waveform in a gradient-echo pulse sequence. The two gradient lobes, the prephasing lobe $G_{x,p}(t)$ and readout lobe $G_x(t)$, have the opposite polarity.

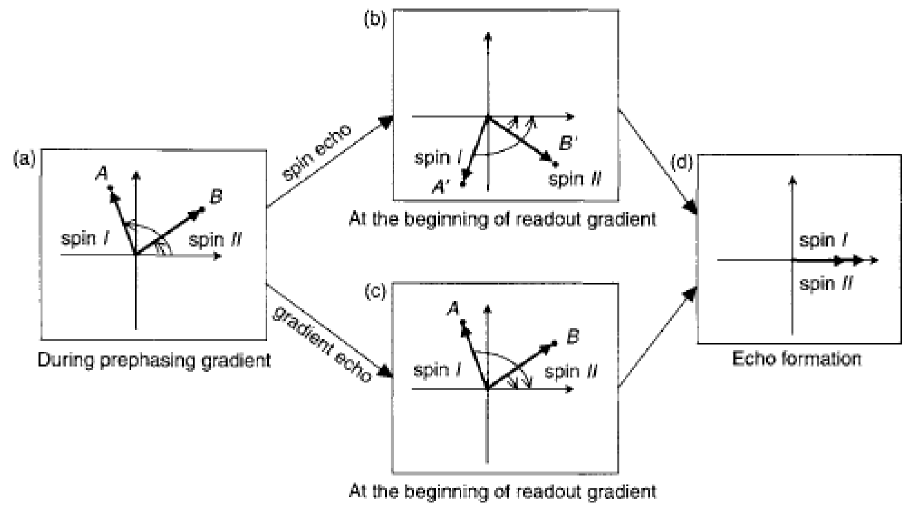
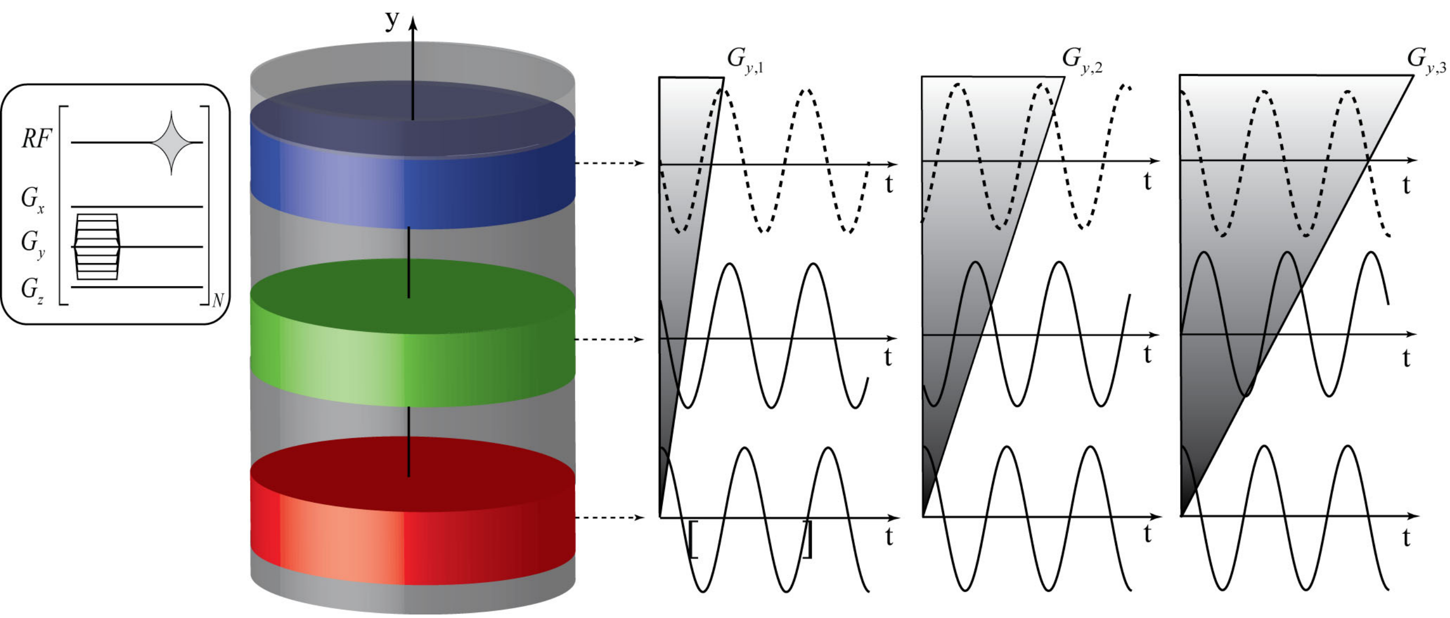


FIGURE 8.4 A diagram showing the effect of prephasing and readout gradients on a spin system consisting of two spin isochromats, I and II. (a) The prephasing gradient introduces a phase dispersion. In a spin-echo pulse sequence, the phase dispersion is reversed by the 180° RF pulse (b). When a readout gradient with the same polarity is applied, the spins I and II continue the phase accumulation in the same direction, producing an echo in (d). In a gradient-echo pulse sequence, the polarity of the readout gradient is the opposite to that of the prephasing gradient. The directions of phase accumulation of the spins are reversed (c), producing an echo as shown in (d).

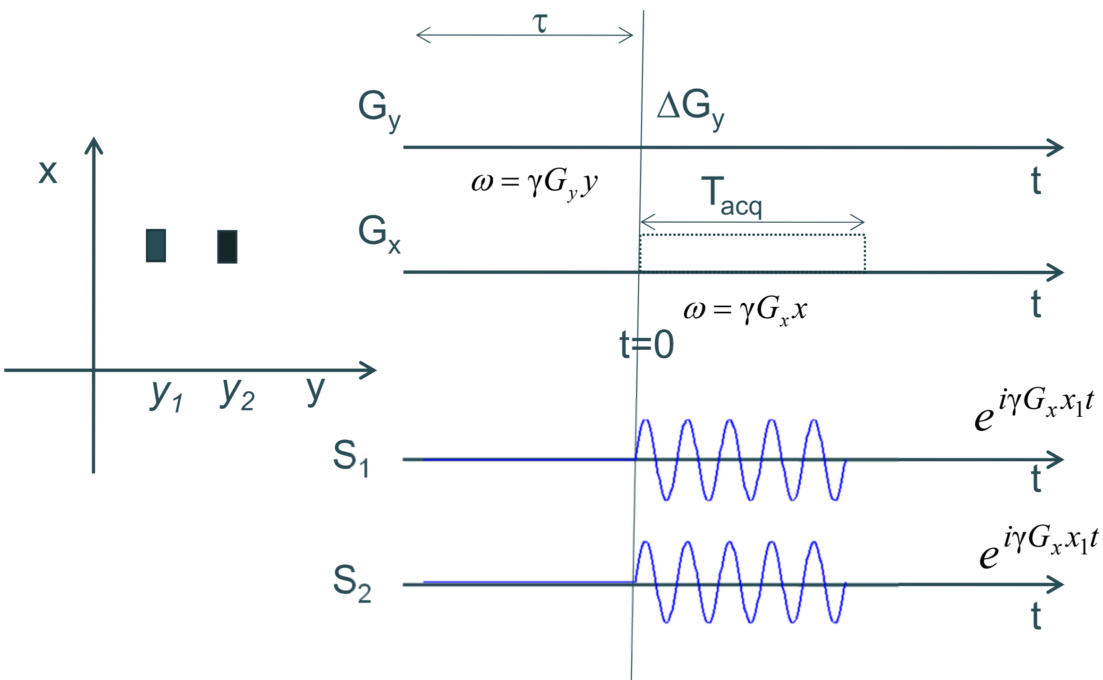
SPATIAL ENCODING – PHASE ENCODING



$$\phi\,(\mathrm{y}) = \gamma \cdot y \cdot G_y \cdot \tau$$

PHASE ENCODING

– In a single acquisition, frequency encoding indicates the position in one dimension. Encoding 2nd and 3rd dimension is usually accomplished via phase encoding (position is encoded in the phase of the NMR signal)



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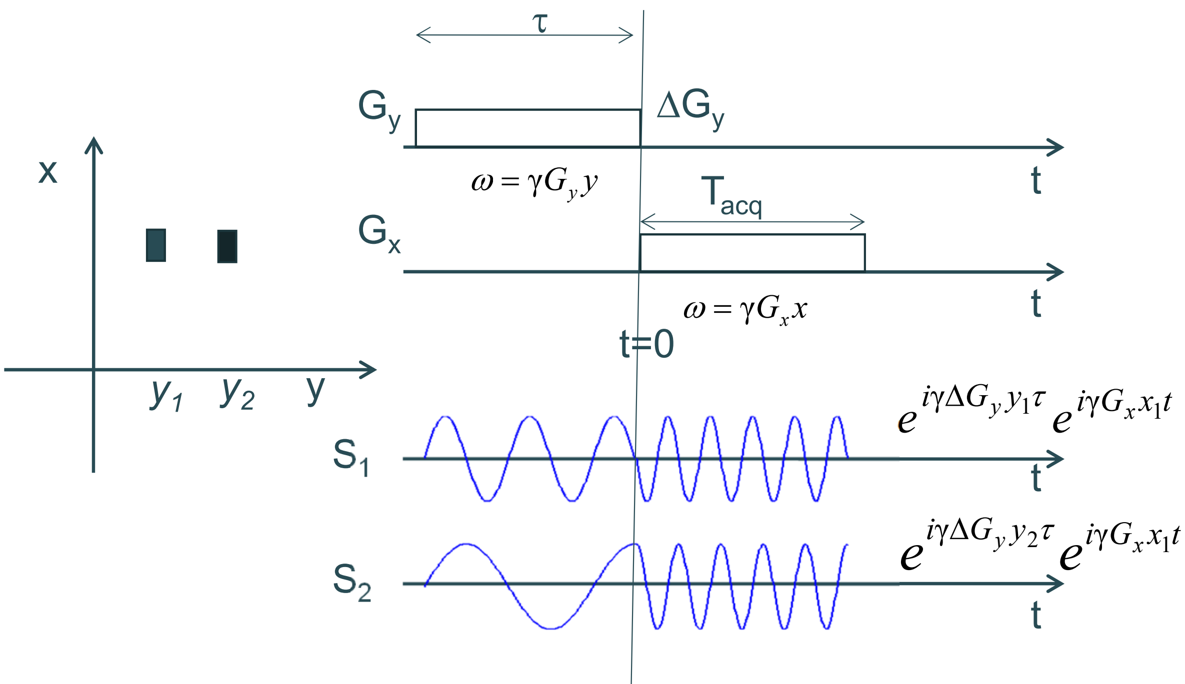

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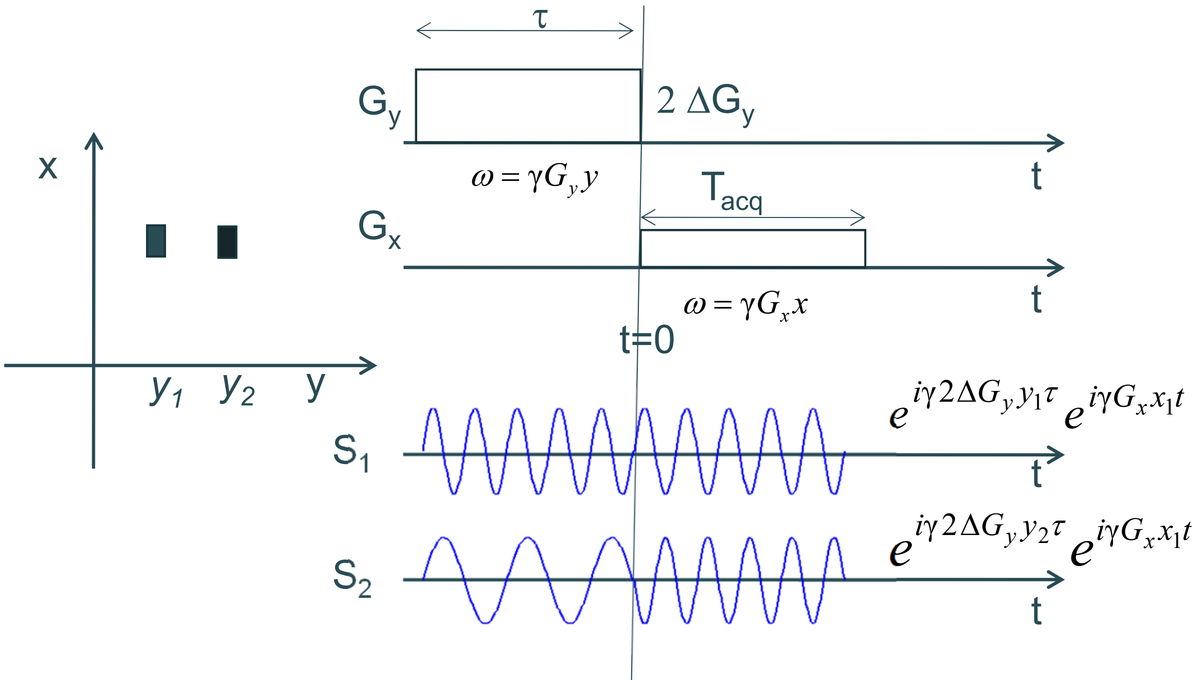


Phase of the signal depends on the y position

while

Frequency depends on the x position

We can not unambiguously identify y from a single measurement…

Multiple measurements are needed.

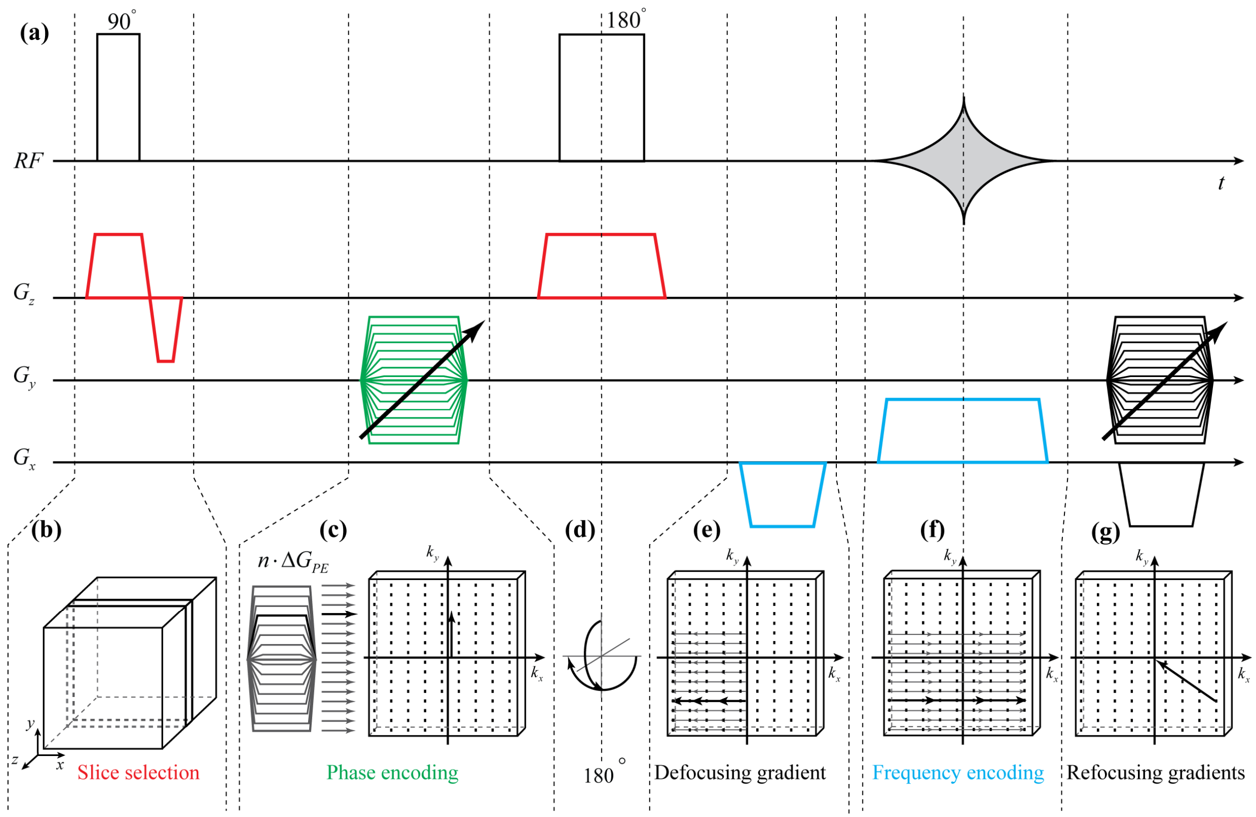
HOW TO ENCODE A 3D OBJECT?

HOW TO ENCODE A 3D OBJECT?

- Conventional 2D acquisitions
- Phase encoding based – (3D conventional acquisition)
- Slice selection based – (spectroscopy localization)

HOW TO ENCODE A 3D OBJECT?

- Conventional 2D acquisition



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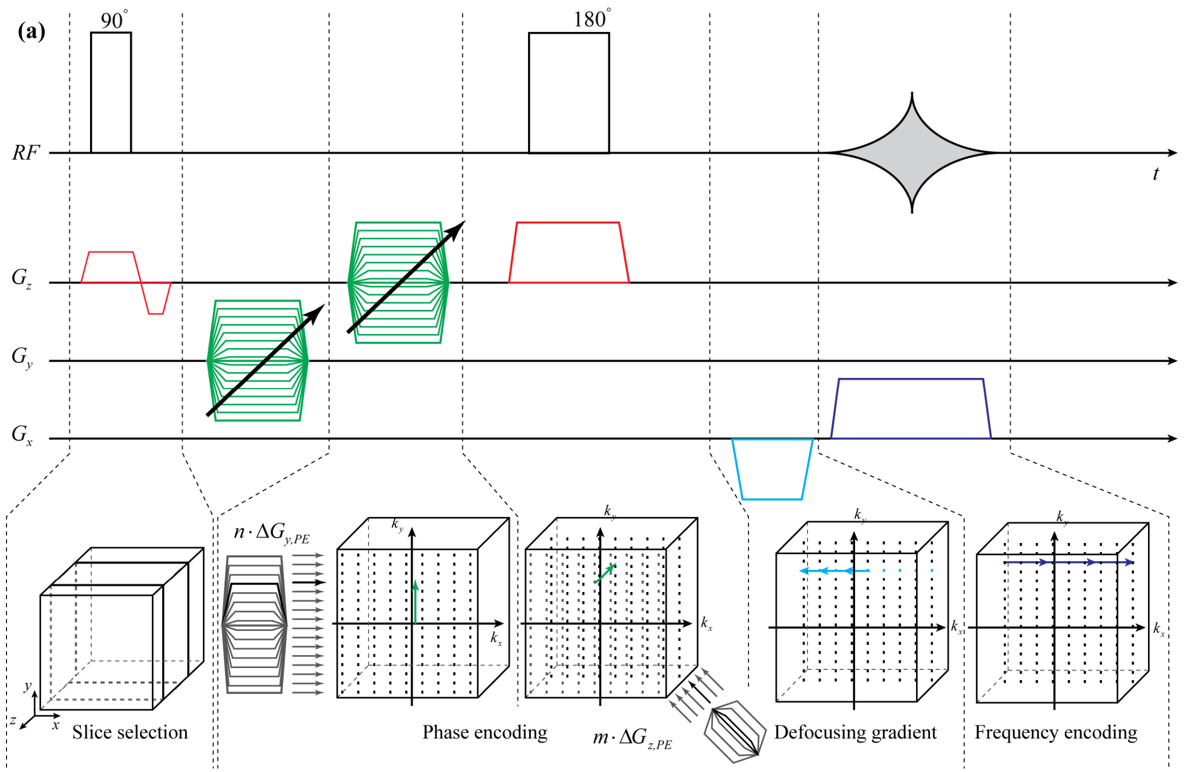
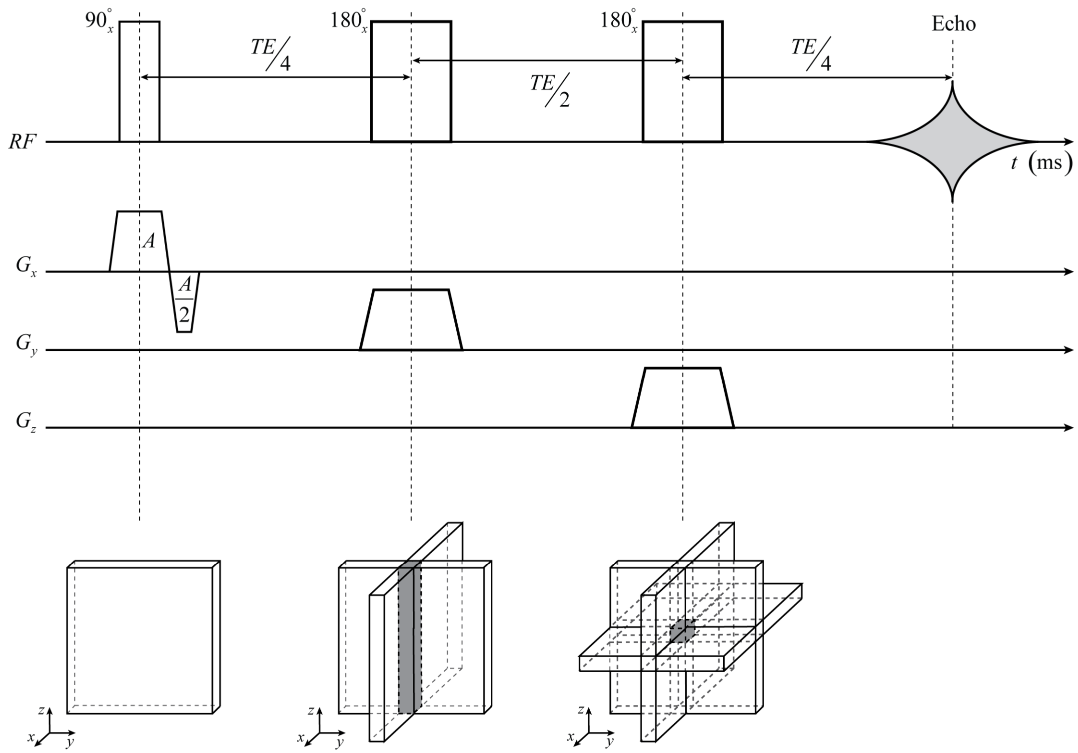
HOW TO ENCODE A 3D OBJECT?

- Phase encoding – Conventional 3D acquisition



HOW TO ENCODE A 3D OBJECT?

- Slice selection – Conventional spectroscopic localization



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SOME NOTIONS ABOUT K-SPACE

SOME NOTIONS ABOUT K-SPACE

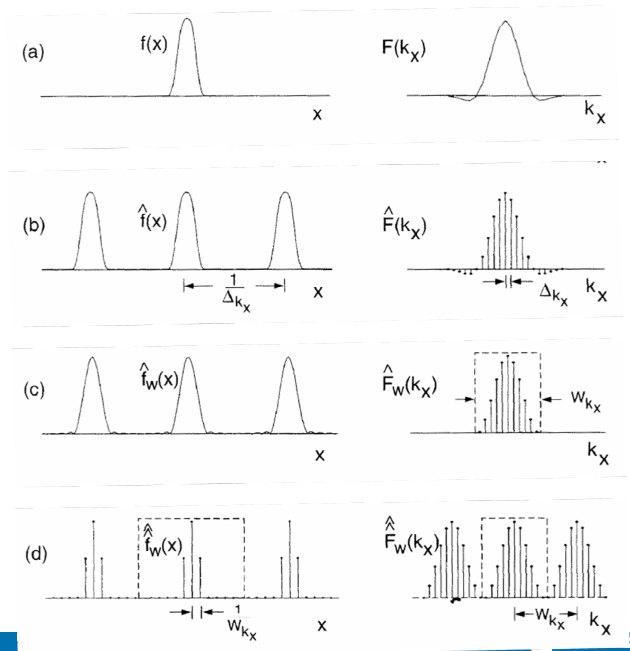
- Some notions about k-space
- Discrete Fourier Transform
- What is where in the k-space

DISCRETE FOURIER TRANSFORM

FT $\longrightarrow$ $G(k) = \int_{-\infty}^{\infty} g(x) e^{-2\pi i k x} dx$

DFT $\longrightarrow$ $G(\frac{p}{L}) = \sum_{q=-n}^{n-1} g\left(\frac{qL}{2n}\right) e^{-2\pi p q/(2n)}$ $\quad p = -n, -n+1 \ldots, n-1 \quad L = 2n\Delta x$

Reconstructed Image Signal sampled



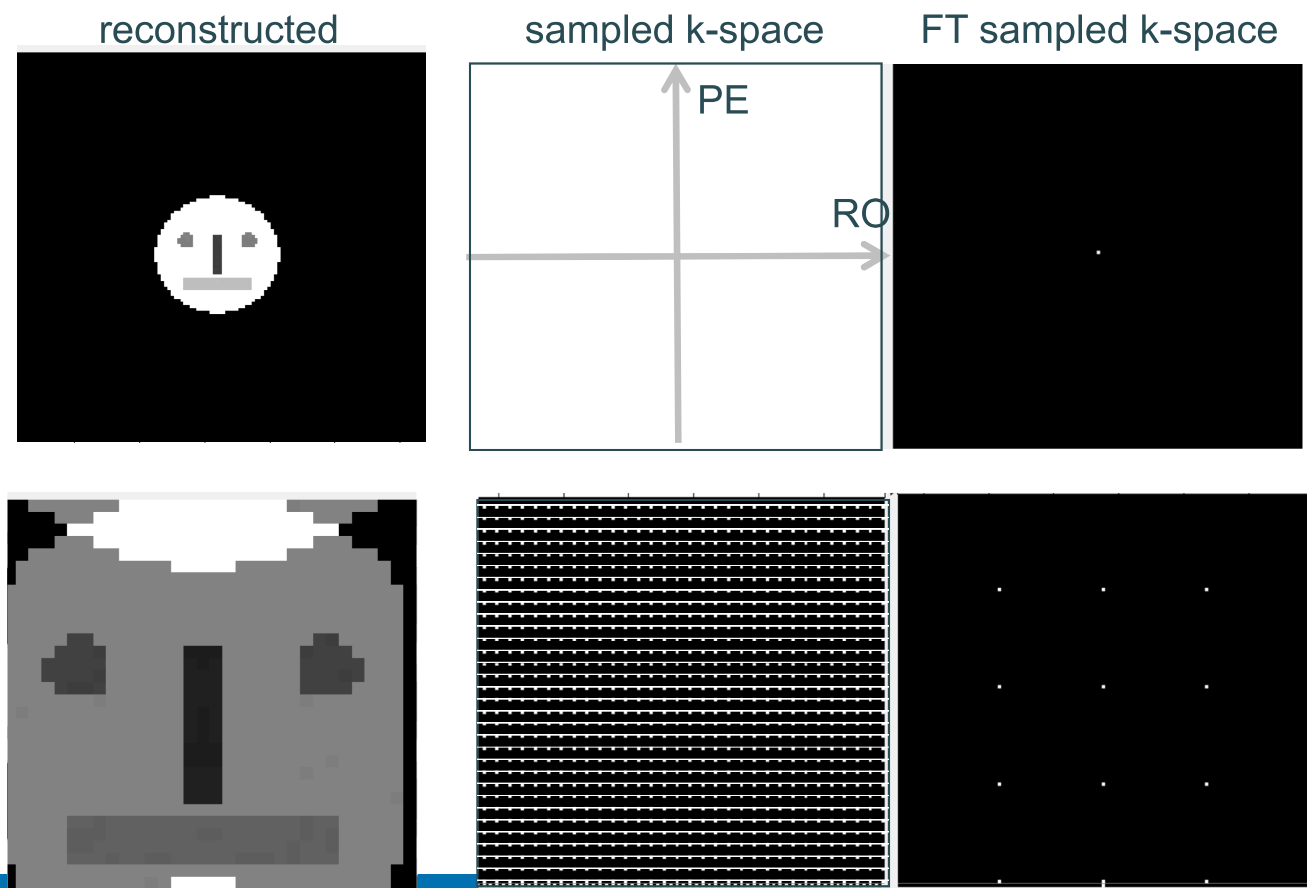
ideally

we can only sample every once in a while

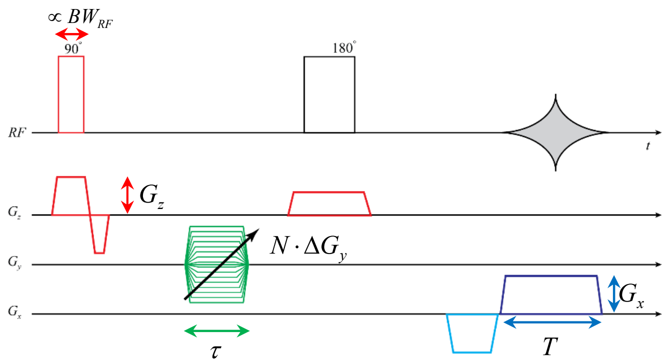
we can only sample for so long

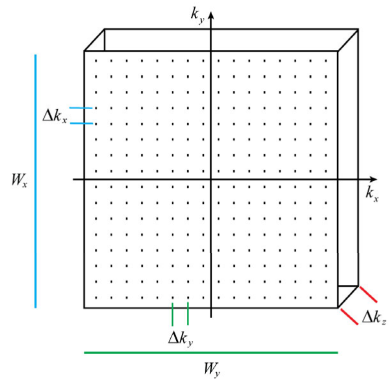
and we will only keep so much information

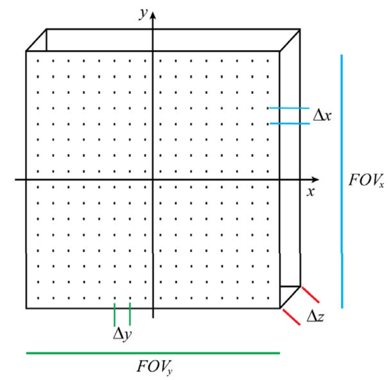
DISCRETE FOURIER TRANSFORM



SOME NOTIONS ABOUT K-SPACE







Slice Selection $$FOV_z = \frac{BW_{RF}}{\gamma G_z}$$

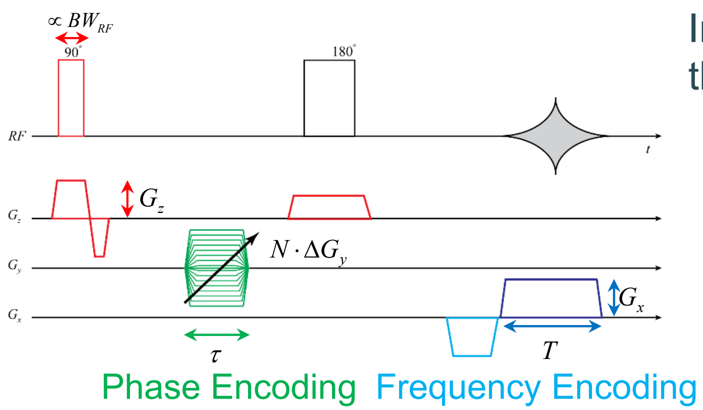
Resolution

Phase Encoding $$FOV_y = \frac{2\pi}{\Delta k_y} = \frac{2\pi}{\gamma \Delta G_y \tau} \qquad \delta y = \frac{2\pi}{W_{k_y}} = \frac{2\pi}{\gamma \Delta G_y N \tau}$$

Frequency Encoding $$FOV_x = \frac{2\pi}{\Delta k_x} = \frac{2\pi}{\gamma G_x \Delta T} \qquad \delta x = \frac{2\pi}{W_{k_x}} = \frac{2\pi}{\gamma G_x T}$$

- Large FOV requires small steps in k
- $\Delta G_y\ \tau$ creates a 2π cycle across the FOV
- High spatial resolution requires large Gradient x time product
- N/2 $\Delta G_y\ \tau$ creates a π shift between two successive pixels

SOME NOTIONS ABOUT K-SPACE



In the rotating frame, neglecting relaxation, the NMR signal can be written as:

$$S(t) = \iiint_{sample} \rho(x, y, z) \cdot dxdydz$$

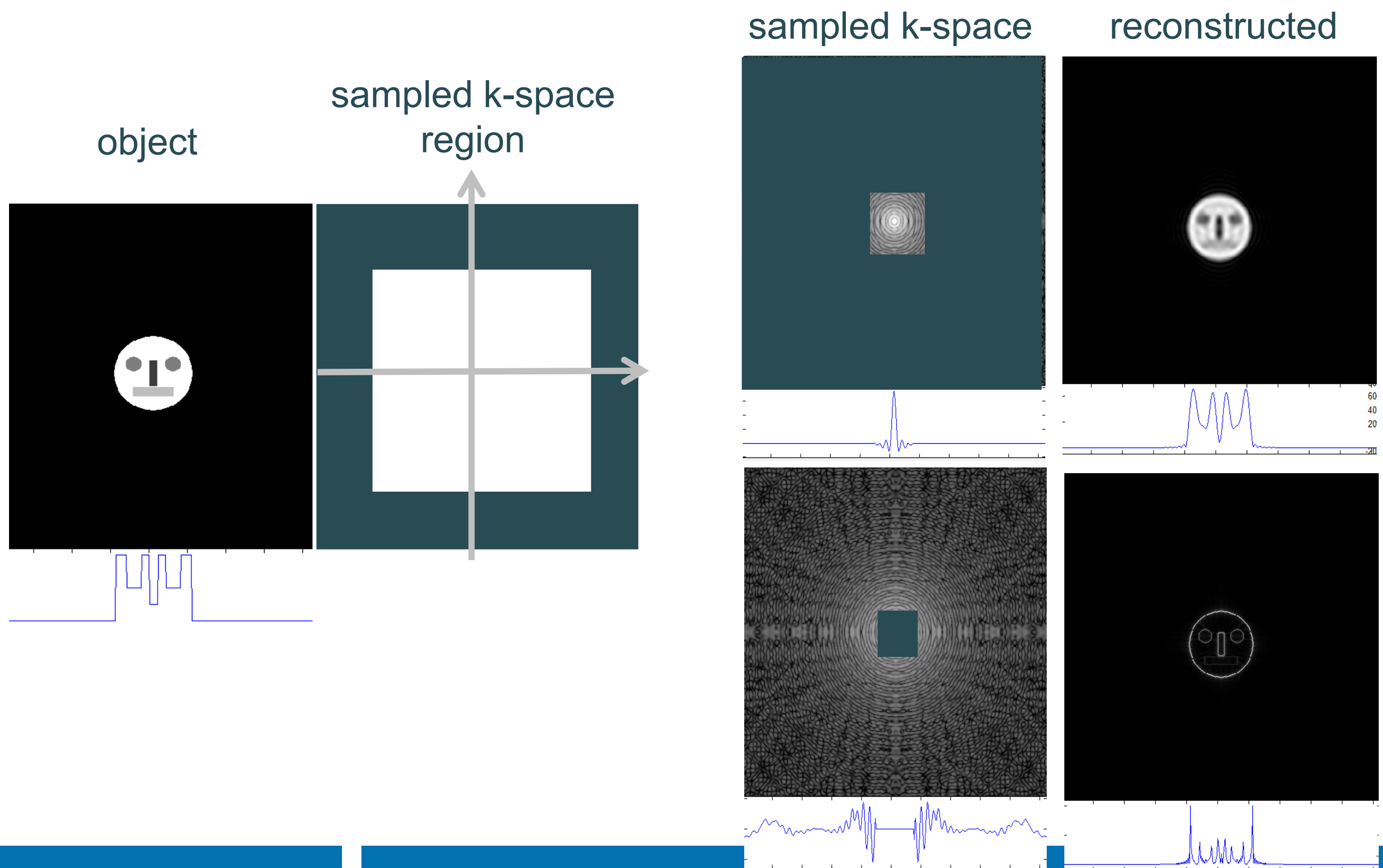
$$S(t) = \iiint_{sample} \rho(x, y, z) \cdot e^{-i\gamma \cdot G_x \cdot t \cdot x} \cdot dxdydz$$

$$\phi(y) = \gamma \cdot n \cdot \Delta G_y \cdot y \cdot \tau$$

$$S^n(t) = \iiint_{sample} \rho(x, y, z) \cdot e^{-i\gamma \cdot G_x \cdot t \cdot x} \cdot e^{-\gamma \cdot n \cdot \Delta G_y \cdot y \cdot \tau} \cdot dxdydz$$

To disentangle phase encoding periodicity *N* measurements should be repeated varying the increment $n\Delta G_y$ (n=-N,-N+1,…0,1,…,N). Image can then be reconstructed taking the 2D discrete Fourier transform of the signals

WHAT IS WHERE IN K-SPACE



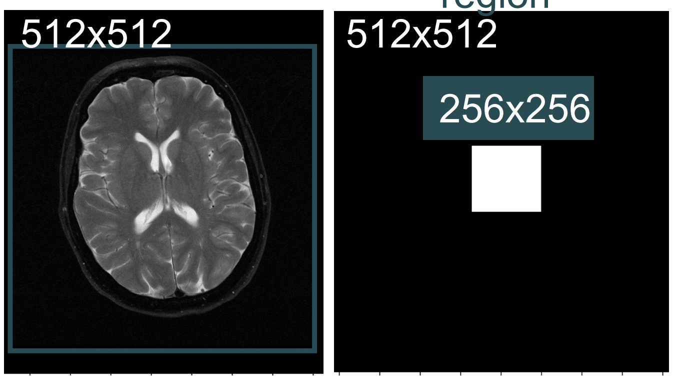
Low spatial frequencies (center of k-space) - **coarse structure**

High spatial frequencies (edge of k-space) - **detail**

WHAT IS WHERE IN K-SPACE

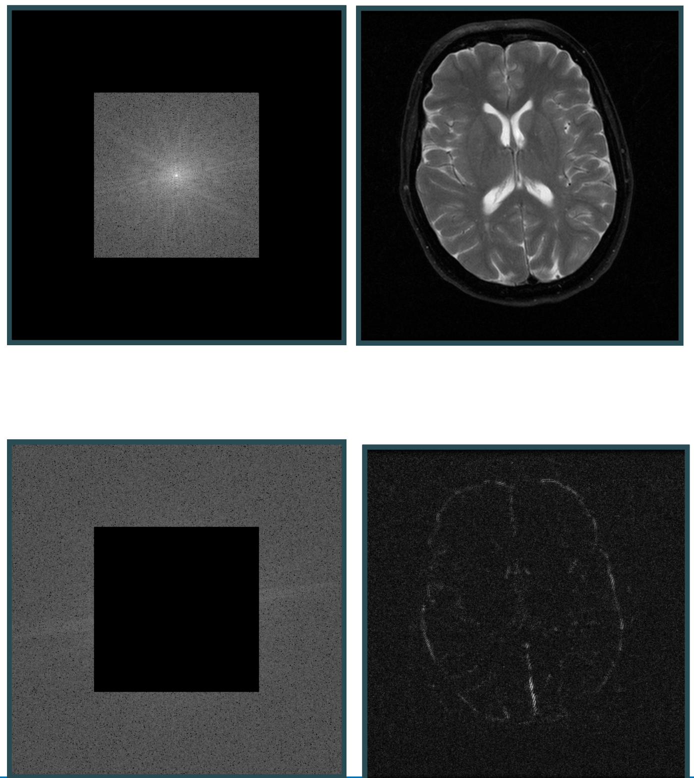
object

sampled k-space region



sampled k-space

reconstructed



Low spatial frequencies (center of k-space) - **coarse structure**

High spatial frequencies (edge of k-space) - **detail**

SUMMARY & ACKNOWLEDGMENTS

MRI is incredibly rich & versatile

Slides:
Nicolas Kunz
Jose Marques
Cristina Cudalbu
Rolf Gruetter

Thank you for listening! Questions?

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THANK YOU FOR YOUR ATTENTION

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